

MATHEMATICAL METHODS UNITS 3 AND 4

2.02 The sum, difference and quotient of two functions.

If f and g are functions of a real variable x , then the sum $f + g$ is defined as follows:

$$(f + g)(x) = f(x) + g(x).$$

The values $f(x)$ and $g(x)$ are defined only if x is in the domain of f and g . $\text{dom}(f + g) = \text{dom } f \cap \text{dom } g$

Similarly,

$$(f - g)(x) = f(x) - g(x). \quad \text{dom}(f - g) = \text{dom } f \cap \text{dom } g$$

$$(fg)(x) = f(x)g(x). \quad \text{dom}(fg) = \text{dom } f \cap \text{dom } g$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{provided } g(x) \neq 0, \text{ dom}(f/g) = \text{dom } f \cap \text{dom } g$$

Example 1

If $f(x) = \sqrt{x-2}$ for all $x \geq 2$ and $g(x) = \sqrt{4-x}$ for all $x \leq 4$, find:

$f + g$

$$\text{dom } f \cap \text{dom } g = [2, 4]$$

$$(f + g)(x) = f(x) + g(x) = \sqrt{x-2} + \sqrt{4-x}$$

$$\text{dom}(f + g) = [2, 4]$$

$(f + g)(3)$

$$(f + g)(3) = \sqrt{3-2} + \sqrt{4-3} = 2$$

$(fg)(x)$

$$(fg)(x) = f(x)g(x) = \sqrt{(x-2)(4-x)}$$

$$\text{dom}(fg) = [2, 4]$$

$(fg)(3)$

$$(fg)(3) = \sqrt{(3-2)(4-3)} = 1$$